An energy-momentum method for ordinary differential equations with an underlying k-polysymplectic structure

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A k-polysymplectic manifold is a pair $(P, \boldsymbol{\omega})$, where P is a manifold, while $\boldsymbol{\omega} \in \Omega^2(P, \mathbb{R}^k)$ is a closed $\mathbb{R}^k$-valued differential 2-form such that $\ker \boldsymbol{\omega} = 0$. The k-polysymplectic Marsden-Weinstein reduction theorem allows for reducing a k-polysymplectic Hamiltonian system, let us say $(P, \boldsymbol{\omega}, \boldsymbol{h})$, admitting a Lie group action $\Phi : G \times P \to P$ leaving a $\mathbb{R}^k$-valued function $\boldsymbol{h}$, where $\iota_X \boldsymbol{\omega} = d\boldsymbol{h}$ for some vector field X and $\boldsymbol{\omega}$ invariant. Particularly, through the Ad^k-equivariant k-polysymplectic momentum map $\mathbf{J}^\Phi : P \to (\mathfrak{g}^*)^k$ induced by Φ, where $\mathfrak{g}$ is the Lie algebra of G, one can find conditions to guarantee the existence of a reduced polysymplectic Hamiltonian system $(\mathbf{J}^{\Phi-1}(\boldsymbol{\mu})/G_{\boldsymbol{\mu}}, \boldsymbol{\omega}_{\boldsymbol{\mu}}, \boldsymbol{h}_{\boldsymbol{\mu}})$, where $G_{\boldsymbol{\mu}}$ is the isotropy subgroup of $\boldsymbol{\mu} \in (\mathfrak{g}^*)^k$ relative to the coadjoint action $\mathrm{Ad}^k : G \times (\mathfrak{g}^*)^k \to (\mathfrak{g}^*)^k$ and $\boldsymbol{h}_{\boldsymbol{\mu}}$ is a $\mathbb{R}^k$-valued function on $\mathbf{J}^{\Phi-1}(\boldsymbol{\mu})/G_{\boldsymbol{\mu}}$ whose pull-back to $\mathbf{J}^{\Phi-1}(\boldsymbol{\mu})$ retrieves the value of $\boldsymbol{h}$ on it.

Our poster presents an extension of the classical energy-momentum method to k-polysymplectic manifolds for the study by means of $\boldsymbol{h}$ the stability close to the equilibrium points of a reduced k-polysymplectic Hamiltonian system $(\mathbf{J}^{\Phi-1}(\boldsymbol{\mu})/G_{\boldsymbol{\mu}}, \boldsymbol{\omega}_{\boldsymbol{\mu}}, \boldsymbol{h}_{\boldsymbol{\mu}})$, i.e. points in P where the projection of the vector field X vanishes, and the analysis of solutions of $(P, \boldsymbol{\omega}, \boldsymbol{h})$ close to points in P that project onto equilibrium points of $(\mathbf{J}^{\Phi-1}(\boldsymbol{\mu})/G_{\boldsymbol{\mu}}, \boldsymbol{\omega}, \boldsymbol{h}_{\boldsymbol{\mu}})$ via the projection $\pi_{\boldsymbol{\mu}} : \mathbf{J}^{\Phi-1}(\boldsymbol{\mu}) \to \mathbf{J}^{\Phi-1}(\boldsymbol{\mu})/G_{\boldsymbol{\mu}}$. Moreover, we will explain that the Ad^{*k}-equivariance property of a k-polysymplectic momentum map $\mathbf{J}^\Phi$ may be omitted because of the existence of the so-called affine action $\boldsymbol{\Delta} : G \times (\mathfrak{g}^*)^k \to (\mathfrak{g}^*)^k$, ensuring $\boldsymbol{\Delta}$-equivariance of $\mathbf{J}^\Phi$.