Talks on the Conference Geoquant 2015, CIMAT

September 14 – 18, 2015

(Version 28.8.2015)

1. **Luis Alvarez-Consul (ICMAT, Madrid, Spain)**

   Title: Gravitating vortices, cosmic strings and the Kähler-Yang-Mills equations

   Abstract: I will start with a review of the relation between vortices on a Riemann surface and instantons on a Kähler surface via dimensional reduction. I will then go on to study gravitating vortices and their relation via dimensional reduction to certain equations, with a symplectic origin, coupling Kähler metrics and Yang-Mills connections. Finally I will use their symplectic interpretation to obtain obstructions to the existence of gravitating vortices. I will also mention the physical meaning of gravitating vortices in cosmology and the relation to previous work in physics. Joint work with M. Garcia-Fernandez and O. Garcia-Prada.

2. **Jorgen Ellegard Andersen (QGM, Aarhus University, Denmark)**

   Title: TBA

   Abstract:

3. **Indranil Biswas (TIFR, India)**

   Title: Bohr-Sommerfeld Lagrangians of moduli spaces of Higgs bundles

   Abstract: Let X be a compact connected Riemann surface of genus at least two. Let $M_H(r, d)$ denote the moduli space of semistable Higgs bundles on X of rank r and degree d. We prove that the compact complex Bohr-Sommerfeld Lagrangians of $M_H(r, d)$ are precisely the irreducible components of the nilpotent cone in $M_H(r, d)$. This generalizes to Higgs G-bundles and also to the parabolic Higgs bundles. (Joint work with Niels Leth Gammelgaard and Marina Logares.)

4. **Elena Bunkova (Steklov Institute of Mathematics, Moscow, Russia)**

   Title: Formal group for elliptic function of level 3

   Abstract: I will give a solution to the long-standing problem: find the form of the formal group whose exponent is the elliptic function of level 3. I will introduce new results on the coefficient ring of this group and describe its relations with known universal formal groups.

5. **Mirek Englis**

   Title: Orthogonal Polynomials, Laguerre Fock Space and Quasi-classical Asymptotics

   Abstract: We describe an unorthodox variant of the Berezin-Toeplitz quantization scheme associated with the Laguerre polynomials and making connection with a variant of the Fock space which houses the familiar Barut-Girardello coherent states, and exhibit the (somewhat surprising) semiclassical asymptotics of the associated Toeplitz operators, as well as a new "squeezing" operator mapping unitarily the standard monomial bases into the Laguerre polynomials. Further extension to the case of Legendre polynomials is likewise discussed. [Joint work with S.-T. Ali (Montreal).]

6. **Anton Fonarev (Steklov Mathematical Institute, Moscow, Russia)**

   Title: Derived categories of curves as components of Fano varieties

   Abstract: A well known result of Beilinson gives an amazingly simple description of the bounded derived category of coherent sheaves on the projectile line. However, a smooth projective curve C of positive genus does not admit nontrivial semi-orthogonal decompositions. An interesting question is whether such a category can be realised as a semi-orthogonal component of a smooth Fano variety. In particular, it is conjectured that D(C) embeds fully and faithfully into the derived category of the moduli space of rank 2 stable vector bundles on C. We are going to prove this conjecture for a generic curve C. Based on a joint work with Alexander Kuznetsov.

7. **Hajime Fujita**

   Title: *TBA*

8. **Tomas Gomez (ICMAT, Madrid, Spain)**

   Title: Torelli theorem for the parabolic Deligne-Hitchin moduli space

   Abstract: The Deligne-Hitchin moduli space is a partial compactification of the moduli space of λ-connections. It includes as closed subvarieties the moduli spaces of Hitchin bundles ($\lambda = 0$) and of holomorphic connections ($\lambda = 1$), exhibiting the later as a deformation of the former. It can also be interpreted as the twistor space for the moduli space of Hitchin bundles. We show a Torelli theorem for a parabolic verion of this moduli space (joint work with David Alfaya).

9. **Peter Gothen (Centro de Matemtica da Universidade do Porto, Portugal)**
   Title: Birationality of moduli of U(p,q)-Hitchin pairs

   Abstract: U(p,q)-Hitchin pairs are twisted quiver bundles for a quiver with two vertices and two arrows connecting them, one in each direction. As such, the natural stability condition for U(p,q)-Hitchin pairs on curves depends on a real parameter. We analyse wall crossing as the stability parameter varies. The fact that the quiver has a loop introduces new phenomena, not present in the previously studied cases of triples and chains. From our results we can deduce birationality results for the moduli spaces in a certain parameter range. This talk is based on joint work with Azizeh Nozad

10. **Tomoyuki Hisamoto (Nagoya University, Japan)**

    Title: On uniform K-stability

    Abstract: It is a joint work with Sbastien Boucksom and Mattias Jonsson. We introduce functionals on the space of test configurations, as non-Archimedean analogues of classical functionals on the space of Kaeler metrics. Then, uniform K-stability is defined as a counterpart of K-energy's coercivity condition. Finally, we study uniform K-stability especially in the case of Kähler-Einstein manifolds

11. **Alexander Karabegov (Abilene Christian University, USA)**

    Title: On the phase form of a deformation quantization with separation of variables

    Abstract: Given a star product with separation of variables on a pseudo-Kaehler manifold, we obtain a new formal (1,1)-form from its classifying form and call it the phase form of the star product. The cohomology class of a star product with separation of

variables equals the class of its phase form. We show that the phase forms can be arbitrary and they bijectively parametrize the star products with separation of variables. We also describe the action of a change of the formal parameter on a star product with separation of variables, its formal Berezin transform, classifying form, phase form, and canonical trace density. Informally, this construction gives an algebraic justification of the following statement. Given a 1/h-oscillatory unitary integral transform, the full asymptotic expansion as h approaches zero of the exponent of its integral kernel is completely determined by its imaginary part, the phase.

12. **Akishi Kato (Tokyo University, Japan)**

    Title: Quiver mutation loops and partition q-series

    Abstract: Quivers and their mutations are ubiquitous in mathematics and mathematical physics; they play a key role in cluster algebras, wall-crossing phenomena, gluing of ideal tetrahedra, etc. Recently, we introduced a partition q-series for a quiver mutation loop (a loop in a quiver exchange graph) using the idea of state sum of statistical mechanics. The partition q-series enjoy some nice properties such as pentagon move invariance. We also discuss their relation with combinatorial Donaldson-Thomas invariants, as well as fermionic character formulas of certain conformal field theories. This is a joint work with Yuji Terashima (Tokyo Institute of Technology).

    Reference: arXiv:1403.6569 [Comm.Math.Phys. 336(2015)811-830] and arXiv:1408.0444 [Comm.Math.Phys. to appear].

13. **Sergei Lando (HSE Moscow, Russia)**

    Title: Computation of universal polynomials for characteristic classes of singularities

    Abstract:

    Any space of meromorphic functions is stratified according to the number of critical values of these functions. The cohomology classes Poincare dual to the strata of such a stratification can be described in terms of certain universal classes, namely, the relative Chern classes of mappings. The standard tool for the description are Thom polynomials. Their computation is a complicated problem, which usually is done step-by-step and requires the knowledge of a complete classification of singularities. However, in some cases computation of characteristic classes for spaces of meromorphic functions on algebraic curves can be done explicitly for large series of classes. Certain corresponding generating functions are solutions to integrable hierarchies. Further computations pose interesting problems of both geometric and combinatorial nature.

    The talk is based on a joint work with Maxim Kazarian and, partly, Dimitri Zvonkine

14. **Jan Manschot (Trinity College, Dublin, Ireland)**

    Title: Sheaves on surfaces and generalized Appell functions

    Abstract: I will discuss generating functions of topological invariants of moduli spaces of semi-stable sheaves over a complex surface. For rational and ruled surfaces, these generating functions can be explicitly determined for arbitrary rank, Chern classes and polarization. To classify and study the building blocks of these generating functions, the notion of Appell functions with signature (m,n) is introduced. For n=1, these functions reduce to the known class of Appell functions with multiple variables or higher level.

15. **Andrey E. Mironov (Sobolev Institute of Mathematics, Novosibirsk, Russia)**

    Title: Commuting ordinary differential operators with polynomial coefficients and automorphisms of the first Weyl algebra

    Abstract: We discuss new results on commuting ordinary differential operators of rank two. In particular, we point out a relation between eigenfunctions of one-dimensional Schrodinger operator with polynomial potentials of degree 3, 4, and eigenfunctions of rank two commuting ordinary differential operators. We study also an action of the group of automorphisms of the first Weyl algebra on commuting ordinary differential operators with polynomial coefficients. We show that in the case of spectral curves of genus one the space of orbits is infinite. The results were obtained with B.T.Saparbaeva and A.B.Zheglov. The above results are closely related to the Dixmier conjecture, and further on (by Kanel-Belov and Kontsevich), to the Jacobian conjecture. In my talk, I am going comment on these relations.

16. **Motohiko Mulase (UCD, Davis, USA)**

    Title: Topological recursion, 2D TQFT, and quantization of Hitchin spectral curves.

    Abstract: The talk is aimed at giving a mathematical introduction to the topological recursion, an inductive mechanism to solve certain enumerative geometry problems, due to Eynard, Orantin, and others, originated in physics. The nature of the recursion is explained in analogy of 2D TQFT. Then the procedure is generalized to Hitchin spectral curves for their quantization. Several concrete examples of the quantum curves will also be presented.

17. **Andre Oliveira (UTAD and University of Porto, Portugal)**

    Title: Geometry of quadratic pairs moduli spaces

    Abstract: A quadratic pair on a compact Riemann surface X is a generalisation of an orthogonal bundle over X. One motivation for the study of the moduli spaces of these objects comes from Higgs bundles for the real symplectic group $\mathrm{Sp}(2n, \mathbb{R})$, hence from representations of $\pi_1 X$ in $\mathrm{Sp}(2n, \mathbb{R})$. We will explain this motivation and present some basic results on the geometry and topology of the moduli spaces of rank 2 quadratic pairs.

18. **Yuji Sano (Kumamoto University, Japan)**

    Title: Quantization of extremal metrics and the modified K-energy

    Abstract: I will introduce a modification of balanced metrics to approximate extremal metrics by them. As an application, it implies the fact that the extremal metrics attains the lower bound of the modified K-energy. This is a joint work with Carl Tipler.

19. **Andrei I. Shafarevich (Moscow State University, Russia)**

    Title: Lagrangian manifolds and Maslov indices corresponding to the spectral series of Schrödinger operators with delta-potentials on symmetric manifolds.

    Abstract: We study spectral series of the Schrödinger operator with delta-type potential on 2D or 3D Riemannian spherically symmetric manifold. Lagrangian manifolds, corresponding to these series, do not coincide with Liouville tori. We describe topological structure of these manifolds as well as Maslov indices, entering quantization conditions.

In particular, we study the effect of the jump of the Maslov index via passing through the critical values of the multipliers of the delta-functions.

20. **Oleg K. Sheinman (Steklov Mathematical Institute, Russia)**

    Title: Lax operator algebras, Hitchin systems, spinning tops, etc.

    Abstract: Lax operator algebras appear as a new class of current algebras on Riemann surfaces, naturally generalizing loop algebras, and closely related to finite-dimensional integrable systems, in particular listed in the title. The same conditions make certain meromorphic Lie algebra-valued functions to form an infinite-dimensional Lie algebra, and to be Lax operators, yielding commuting Hamiltonian hierarchies, in particular, the celebrated Hitchin Hamiltonians. With the notion of Lax operator algebras, we can treat the theory of finite-dimensional integrable systems in a very general set-up of arbitrary semi-simple Lie algebras, root systems, invariant polynomials, and related objects. The notion of Lax operator algebras goes back to I.Krichever (2001), I.Krichever and the author (2007), and in full generality invented by the author in 2014 (thanks to discussions with E.B.Vinberg).

    In the talk, we will give the definition of Lax operator algebras, introduce Lax equations with spectral parameter on a Riemann surface (Zakharov, Mikhailov, 1983), formulate the existence of commuting hierarchies, and give a construction of their Hamiltonians via invariant polynomials of Lax operators.

21. **Siye Wu**

    Title: *TBA*

22. **Hao Xu (University of Pittsburg, USA)**

    Title: Asymptotics of Bergman kernel and deformation quantization

    Abstract: The asymptotic expansion of the heat and Bergman kernels have numerous applications in spectral geometry and complex geometry respectively. The asymptotic expansion of deformation quantization provides rules for deforming the commutative algebra of classical observables to a noncommutative algebra of quantum observables. All these asymptotic coefficients encode important geometric information. We will discuss an expression of these coefficients as a summation over strongly connected graphs and potential applications.