Orderings and Groups, 12-16 June 2023, ICMAT Abstracts

Schedule

	Lunes 12	Martes 13	Miercoles 14	Jueves 15	Viernes 16
10-11	Rivas	Baik	Navas	Lodha	Darbinyan
11-11:30	Coffee Break				
11:30-12:30	Sunic	Chen	Eyard-Bontemps	Steenbock	Fournier-Facio
12:30-15:00	Lunch				
15:00-16:00	Matte-Bon	Maruyama		O'Rourke	
16-16:30	Coffee Break			Pausa Café	
16:30-17:30	Triestino	Clay		Wang	

On Monday 12th at 9:50 am there is going to be a registration desk at the entrance of the ICMAT. Everyday, a bus will depart from Hotel Mediodia at 9 am. Please contact Yago Antolín if you plan to take the bus and you are not hosted at Hotel Mediodía. On Monday, Tuesday and Thursday Lunch will be at CBM. Wednesday and Friday lunch will be served at restaurante "El Goloso".

Talks on Monday

- *Introduction to orderable groups*
  Cristóbal Rivas

  This is an introductory lecture to the subject of orderable groups. It will start with the basic definitions of bi-orderability, left-orderability and basic properties. This talk will serve as an introduction to the different topics discussed during the week.

- *Ordering free groups*
  Zoran Šunić

  We provide a summary of recent results on left orders in free groups and free products. In particular, we present some simple constructions of left orders on free groups and free products and indicate where they come from in terms of group actions.

- *A realisation result for moduli spaces of group actions on the line*
  Nicolás Matte-Bon

  Given a finitely generated group G, the possible actions of G on the real line (without global fixed points), considered up to semi-conjugacy, are encoded by the space of orbits of a flow on a compact space naturally associated to G, that we call the Deroin space of G. In joint work with J. Brum, C. Rivas and M. Triestino, we show a realisation result: every expansive flow on a compact metrisable space of covering dimension 1, satisfying some mild additional assumptions, arises as the Deroin space of a finitely generated group. This is proven by identifying the Deroin space of an explicit family of groups acting on suspension flows of subshifts, which is a variant of a construction introduced in joint work with M. Triestino. This result provides a source of examples of finitely generated groups satisfying various new phenomena for actions on the line, related to their rigidity/flexibility properties and to the structure of connected and path-connected components of the space of actions.

- *Actions of solvable groups on the line*
  Michele Triestino

  A group action on the line is $\mathbb{R}$-focal if it preserves a lamination. $\mathbb{R}$-focal actions arise naturally when considering particular families of groups, such as locally moving or solvable groups. We will discuss about this notion, and see how this can be used to understand general actions on the line of solvable groups. This is part of a joint project with Brum, Matte Bon, and Rivas.

Talks on Tuesday

- *Groups acting on the circle with a veering pair of laminations*
  Harry Baik

  We study some circularly orderable groups called laminar groups. Under natural assumptions on the invariant laminations, one can construct a simply connected 3-manifold on which the given group acts on, and consequently show that the given group is a 3-manifold group. This talk is based on the joint work with KyeongRo Kim and Hongtaek Jung.

- *There are no exotic actions of diffeomorphism groups on 1-manifolds*
  Lei Chen

  Let M be a manifold, N a 1-dimensional manifold. Assuming $r \neq \dim(M) + 1$, we show that any non-trivial homomorphism $\rho\colon \mathrm{Diff}^r_c(M) \to \mathrm{Homeo}(N)$ has a standard form: necessarily M is 1-dimensional, and there are countably many embeddings $\phi_i\colon M \to N$ with disjoint images such that the action of ρ is conjugate (via the product of the ϕ_i) to the diagonal action of $\mathrm{Diff}^r_c(M)$ on $M \times M \times \dots$ on $\cup_i \phi_i(M)$, and trivial elsewhere. This solves a conjecture of Matsumoto. We also show that the groups $\mathrm{Diff}^r_c(M)$ have no countable index subgroups

- *Non-extendable invariant quasimorphisms for groups acting on the circle.*
  Shuhei Maruyama

  Let G be a group and N a normal subgroup of G. In this talk, I will discuss G-conjugation invariant quasimorphisms on N that is not extendable to G, especially a construction of such quasimorphisms using actions of G on the circle. This is part of joint work with Morimichi Kawasaki, Mitsuaki Kimura, Takahiro Matsushita and Masato Mimura.

- *Borel complexity and spaces of orderings*
  Adam Clay

  Many classification problems in mathematics (e.g. classifying finitely generated abelian groups or subgroups of Q up to isomorphism) can be modelled by Borel equivalence relations on Polish spaces. The relative complexity of these problems is captured by the notion of Borel reducibility, which roughly allows one to say when one Borel equivalence relation is "more complicated" or "less complicated" than another. In this talk I will introduce these notions, with a particular focus on the case of countable Borel equivalence relations. From there, we will see that the spaces of left-orderings of countable groups, equipped with the natural action, provides an abundant source of natural examples of countable Borel equivalence relations. I will discuss where these natural examples sit in the complexity hierarchy, and some conjectural connections between the structure of left-ordered groups and the position of their corresponding equivalence relation in the complexity hierarchy. This is joint work with Filippo Calderoni.

Talks on Wednesday

- *TBA*
  Andrés Navas

- *Deformations of $\mathbb{Z}^2$-actions in dimension 1*
  Hélène Eynard-Bontems

  (joint with Andrés Navas) Can any pair of commuting diffeomorphisms of a compact 1D manifold be connected to the trivial pair (id,id) via a path of such pairs ? This question plays an important role in the classification of foliations of 3-manifolds by surfaces. It can be asked in any differentiability class, and we will see that the phenomena at play and the techniques involved to answer it highly depend on the regularity, focussing on a new result in the intermediate regularity C^{1+ac} (where ac stands for absolutely continuous).

Talks on Thursday

- *Finitely presented simple torsion free groups of homeomorphisms of $\mathbb{R}$.*
  Yash Lodha

  I will present a construction of finitely presented simple torsion free groups that act by piecewise linear homeomorphisms of $\mathbb{R}$. These are the first such examples, to our knowledge. Moreover, our examples are also of type F_∞ and have infinite cohomological dimension. This is joint work with James Hyde.

- *Embeddings into simple left-orderable groups*
  Markus Steenbock

  We discuss embeddings into finitely generated simple left-orderable groups. What can the overgroup know about the embedded subgroup? We discuss geometric, algebraic and algorithmic properties, such as quasi-isometry, Frattini, the membership problem and the computability of the left-order. This is on joint work with Arman Darbinyan.

- *Bi-orders on the free metabelian groups*
  Wenhao Wang

  A bi-order on a group G is a total, bi-multiplication invariant order. Such an order is regular if the positive cone associated to the order can be recognised by a regular language. In this talk, I will discuss the convex hull of a bi-order on a non-abelian free metabelian group of finite rank. And I will prove that bi-orders on such groups are never regular.

- *Groups of unitriangular matrices of non-discrete rank*
  Shane O'Rourke

  For a given ordered abelian group Ω and field $\mathbf{k}$ of characteristic zero, we define and consider the group $\mathrm{UT}(\Omega, \mathbf{k})$ of unitriangular $\Omega \times \Omega$ matrices; such groups (and their subgroups) generalise the familiar groups $\mathrm{UT}(n, \mathbf{k})$ of finite rank unitriangular matrices as well as McLain's example of a characteristically simple, locally nilpotent group. We report on work on these groups, and consider their properties and connections with other objects such as groups of order-preserving group automorphisms of Hahn groups, centrally ordered groups, (exponential) $\mathbf{k}$ groups, and rings of formal power series. We also describe a type of Lie correspondence between $\mathrm{UT}(\Omega, \mathbf{k})$ and the Lie algebra $\mathfrak{ut}_0(\Omega, \mathbf{k})$ of matrices of the form $M - I$ where $M \in \mathrm{UT}(\Omega, \mathbf{k})$.

Talks on Friday

- *Computable groups and computable group orderings*
  Arman Darbinyan

  An important class of orderable groups are the (left- or bi-orderable) groups that admit computable orders. From the computability point of view it is interesting to investigate when orderable groups admit computable orders. In particular, a question of Downey and Kurtz asks about the existence of computable orderable groups that do not admit computable orders with respect to any group presentation. In my talk I will discuss answers to this question.

- *Finitely presented left orderable monsters*
  Francesco Fournier-Facio

  A left orderable monster is a finitely generated left orderable group all of whose actions on the line are as complicated as possible, in a precise sense. In his 2018 ICM address, Navas asked whether such monsters can exist. Examples were provided by Hyde–Lodha and by Matte Bon–Triestino shortly thereafter: both families involve a delicate construction, the proof that they are monsters is not easy, and the groups are not finitely presentable. I will present a new construction of left orderable monsters which is different in several ways: most importantly the construction is elementary, and the groups are finitely presented (even type F_∞). This is joint work with Yash Lodha and Matt Zaremsky.