On limiting interpolation methods and compactness

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Interpolation theory plays an important role in the study of function spaces, operator theory and approximation theory, among many other fields of mathematics. Many of these applications are based on the real method $(A_0, A_1)_{\theta,q}$ introduced by Lions and Peetre (see [1]), where $0 < \theta < 1$. Limiting methods where θ can take the values 0 or 1 and A_0 is continuously embedded in A_1 (that is, in the ordered case) are also very useful when decomposing spaces by means of simpler ones and in the study of the boundedness of operators between spaces with complicated structures and of singular integrals (see [2,7]).

To be in the ordered case is essential for the techniques used in those papers, but, from the point of view of interpolation theory, it is only a restriction. For this reason, it is natural to study the extension of limiting methods to arbitrary, not necessarily ordered, couples of Banach spaces $\bar{A} = (A_0, A_1)$. In [4,5] the present author and Fernando Cobos suggested an extension of these methods for arbitrary couples that allows one to produce a sufficiently rich theory, and also studied some of its properties.

On the other hand, interpolation of compact operators is a classical question that has attracted the attention of many authors. As concerns the real method, the final result was obtained by Cwickel [6] and Cobos, Kühn and Schonbek [3], who proved that if $T \in \mathcal{L}(\bar{A}, \bar{B})$ and any of its restrictions $T : A_j \to B_j$ $(j = 0, 1)$ is compact, then the interpolated operator $T : (A_0, A_1)_{\theta,q} \to (B_0, B_1)_{\theta,q}$ is also compact. However, things work differently when dealing with limiting methods.

In this talk, we will review the definitions of the mentioned methods, show some examples and study the behaviour of compact operators under these methods.

References

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