The Bohnenblust–Hille inequalities

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The multilinear Bohnenblust–Hille inequality, proved in 1931, asserts that for each positive integer m there is a constant $C_m \geq 1$ such that

$$\left(\sum_{i_1,\ldots,i_m=1}^{N} \left| T(e_{i_1},\ldots,e_{i_m}) \right|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq C_m \left\| T \right\|,$$

for all positive integer N and all m-linear forms T defined in $\ell_\infty^N \times \cdots \times \ell_\infty^N$. The polynomial Bohnenblust–Hille inequality is a similar result, that controls the norm of the coefficients of the homogeneous polynomials. These inequalities were conceived as tools for the investigation of problems related to Dirichlet series but nowadays the Bohnenblust–Hille inequalities play an important role in different areas of mathematics. The control of the constants involved is crucial for the applications. We present recent results that show that, in contrast to the predictions of the last 80 years, the constants C_m present a slow growth (subpolynomial) and the growth of the constants of the polynomial Bohnenblust–Hille inequality, in the case of complex scalars, is subexponential. As an application of this former result we obtain the exact asymptotic behavior of the n-dimensional Bohr radius.