Spaces of holomorphic functions on non-balanced domains

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When U is an open subset of a complex Banach space E, three topologies are usually considered on the space $H(U)$ of all holomorphic functions on U: the compact open topology τ_0, the Nachbin topology τ_ω and the bornological topology τ_δ. It is known that $\tau_0 = \tau_\omega = \tau_\delta$ if E is finite dimensional, while $\tau_0 < \tau_\omega \leq \tau_\delta$ if E is infinite dimensional and several researchers have been interested in characterizing those spaces E such that $\tau_\omega = \tau_\delta$. The first positive result on that problem was obtained by Dineen in 1972. He proved that if E is a Banach space with an unconditional Schauder basis and U is a balanced open subset of E, then $\tau_\omega = \tau_\delta$ on $H(U)$. Soon after, Cœuré proved an analogous theorem for the space $E = L^1[0, 2\pi]$. Finally, in the nineties, Dineen and Mujica independently obtained the most general result about the problem that we are considering: if E is a separable Banach space with the bounded approximation property and U is a balanced open subset of E, then $\tau_\omega = \tau_\delta$ on $H(U)$.
In this talk, we study the coincidence of these topologies in the non-balanced case. Our main result is the following one: if E is a separable Banach space with the bounded approximation property, U is a balanced open subset of E, A is a closed bounded subset of E, $A \subset U$ and $U \backslash A$ is connected, then $\tau_\omega = \tau_\delta$ on $H(U \backslash A)$. This study about the $\tau_\omega = \tau_\delta$ problem strongly depends on the existence of holomorphic extension from $U \backslash A$ to U.