
On the non-commutative neutrix product of distributions

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Distributions, also known as generalized functions, are comparatively new in modern analysis leading to an extension of some properties of functions defined in the classical sense. For instance, some functions may have finite order of derivative whereas all distributions have infinite order. However, some operations of distributions such as composition, multiplication and convolution can be defined only for particular class of distributions. It is still a challenge to give a unified treatment for those operations. In this presentation we will mainly focus on the product of distributions introduced by B. Fisher :

Let f and g be distributions and let $g_n = (g * \delta_n)(x)$, where $\delta_n(x)$ is a certain sequence converging to the Dirac delta function. The non-commutative neutrix product $f \circ g$ of f and g is defined to be the limit of the sequence $\{fg_n\}$, provided its limit h exists in the sense that

$$N - \lim_{n \rightarrow \infty} \langle f(x)g_n(x), \varphi(x) \rangle = \langle h(x), \varphi(x) \rangle$$

for all functions φ in \mathcal{D} . Some results are proved.

Joint work with Somsak Orankitjaroen.