Compact operators and approximation spaces

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The theory of approximation spaces is a useful and flexible tool which allows to study not only problems in function spaces, but also in spaces of operators and sequence spaces. Given a quasi-Banach space X and scalar parameters $0 < \alpha < \infty, 0 < p, q \leq \infty, \gamma \in \mathbb{R}$, the classical theory deals with spaces X_p^α, and the limiting theory with $X_q^{(0,\gamma)}$. Outstanding examples of spaces X_p^α are Besov spaces $B_{p,q}^s$, Lorentz sequence spaces $\ell_{p,q}$, and the spaces of operators $\mathfrak{L}_{p,q}^{(a)}(E, F)$ consisting of all bounded linear operators between the Banach spaces E, F whose approximation numbers belong to $\ell_{p,q}$ (see [5]). Examples of $X_q^{(0,\gamma)}$ spaces are Besov spaces $B_{p,q}^{0,\gamma}$ with smoothness close to zero, and the Lorentz-Zygmund operator spaces $\mathfrak{L}_{\infty,q,\gamma}^{(a)}(E, F)$ (see [2–4]). In this talk, we investigate compact operators between approximation spaces, paying special attention to the limit case. Applications are given to embeddings between Besov spaces.

References

[1] F. Cobos, O. Domínguez, and A. Martínez, *Compact operators and approximation spaces* (2013). Preprint.

[2] F. Cobos and M. Milman, *On a limit class of approximation spaces*, Numer. Funct. Anal. Optim. **11** (1990), 11–31.

[3] R.A. DeVore, S.D. Riemenschneider, and R.C. Sharpley, *Weak interpolation in Banach spaces*, J. Funct. Anal. **33** (1979), 58–94.

[4] F. Fehér and G. Grässler, *On an extremal scale of approximation spaces*, J. Comp. Anal. Appl. **3** (2001), 95–108.

[5] A. Pietsch, *Approximation spaces*, J. Approx. Theory **32** (1981), 115–134.

Joint work with F. Cobos and A. Martínez.