Sard theorems for Lipschitz functions

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Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a C^k-smooth function. A value $r \in \mathbb{R}^m$ is called critical, if there exists $x \in \mathbb{R}^n$ such that $f(x) = r$ and the derivative $Df(x)$ is not surjective. In this setting, the classical Sard theorem asserts that the set of critical values has Lebesgue measure zero, provided $k > n-m$. Although this result is known to be sharp in the general setting, we shall present nonsmooth versions based on favorite subclasses of Lipschitz continuous functions. (A point x is called Clarke critical for f, if the Clarke generalized Jacobian of f at x contains a matrix $n \times m$ of rang less than m.) The particular case $m = 1$ corresponds to a generalized version of the Morse-Sard theorem.

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