
Some new aspects of James' weak compactness theorem

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We discuss variations of James' weak compactness theorem along two different lines.

First, we introduce two measures of weak non-compactness Ja_E and Ja that quantify, via distances, the idea of boundary behind James' compactness theorem. These measures tell us, for a bounded subset C of a Banach space E and for given $x^* \in E^*$, how far from E or C one needs to go to find $x^{**} \in \overline{C}^{w^*} \subset E^{**}$ with $x^{**}(x^*) = \sup x^*(C)$. A quantitative version of James' compactness theorem is proved using Ja_E and Ja .

Second, we present cases where the thesis of James' weak compactness theorem holds from the information a priori that only some of the functionals of E^* achieve the norm.

Joint work with O. Kalenda, J. Orihuela, and J. Spurný.